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Quantum field theory and unification in AdS5

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Abstract: We consider gauge bosons in the bulk of AdS_5 in a two-brane theory that addresses the hierarchy problem. We demonstrate that one can do a perturbative calculation above the IR scale associated with the second brane. We show such a theory can be consistent with gauge coupling unification at a high scale. We discuss subtleties in this calculation and show how to regulate consistently in a bounded AdS_5 background. Our regularization is guided by the holographic dual of the calculation.

Keywords: Field Theories in Higher Dimensions, AdS-CFT Correspondance, GUT.

JHEP11(2001)003

Contents

1. Introduction **1**

2. Setup **3**

3. Effective four-dimensional theory **3**

4. 5D Position/momentum space propagators **7**
4.1 R_ξ Gauges 7
4.2 Solving the Green's functions 9

5. Feynman rules **10**

6. Limits of the Green's functions **11**

7. Regulating 5D loops **14**

8. Corrections to the radion potential **16**

9. Gauge boson self-energy **17**
9.1 Background field lagrangian 17
9.2 Functional determinants 19
9.3 Numerical results for $I(\Lambda, q)$ 21

10. Coupling constant unification **23**

11. Conclusions **26**

JHEP11(2001)003

1. Introduction

In AdS_5, the coupling for bulk gauge bosons runs logarithmically, not as a power law. For this reason, one can preserve perturbative unification of couplings. Depending on the cutoff, this can occur at a high scale. We show that although it is difficult to do this calculation in a four-dimensional field theory, one can do it in the full five-dimensional context. In this paper, we consider the running of bulk gauge bosons with energy. We discuss several subtleties in the quantity to be calculated, and in the regularization scheme. Our scheme is based on consistency with the holographic

correspondence. We will find that generically, as in the standard model, the couplings almost unify, if the U(1) is normalized consistently with a GUT group as in SU(5) [1], as might be the case under various assumptions for the fundamental physics. For specific choices of cutoff and number of scalar multiplets, there is good agreement with the measured couplings and the assumption of high scale unification.

This addresses one of the apparent weaknesses of the warped extra-dimensional theory that addresses the hierarchy problem, that it appears that one must abandon unification of couplings. This problem has been addressed in the context of large extra dimensions in [2] and [3]. However, even if these mechanisms were to work, one would never have high scale unification. In this paper, we show that the warped scenario that naturally generates the hierarchy [4] also naturally accommodates unification, due to the logarithmic running of the couplings. The scale at which unification occurs depends strongly on the cutoff however; fortunately this is connected to a potentially observable quantity, the number of KK modes.

The calculation we do is similar in some respects to that in [5], in which Pomarol considered bulk gauge boson running but with the assumption that the quarks and leptons live on the Planck brane. He made the nice observation that unification can occur in this scenario. Here, we consider theories that address the hierarchy through the warped geometry. We also assme the existence of a Higgs on the TeV brane to generate weak scale symmetry breaking. Another essential difference in our calculation is in the regularization scheme. We will discuss problems with the effective theory calculation and Pauli Villars regularization. In fact, we will argue that a specific regularization scheme is dictated by the holographic correspondence to four-dimensional field theory.

Our analysis also differs from that suggested in [6]. As we will discuss, the particular quantity we are interested in, the running of the coupling of the massless four-dimensional mode, can only be calculated explicitly from the holographic perspective in a theory in which the TeV brane is explicit. The large tree level logarithmic running does not apply to the running of the zero mode coupling.

The organization of this paper is as follows: first, we introduce the theory and review some of the results of the effective 4D picture. We explore some difficulties with the KK picture for the high scale calculation required for running the couplings. In section 4, we derive the position/momentum space propagators in the R_ξ gauges. We then present some of the Feynman rules and in section 6 we explore the position/momentum space Green's functions in various limits. We introduce and motivate our regularization scheme in section 7. Through some toy calculations we show that it is necessary to modify the boundary conditions and renormalize the Green's functions for energies greater than T in addition to introducing a position dependent cutoff. We then set up an illustrative calculaton, the contribution of a scalar field to the vacuum energy. In section 9, we calculate the 1-loop β-functions. We use the background field method, where the Ward identities are manifest. Finally,

JHEP11(2001)003

section 10 explores unification in two specific examples. We conclude that coupling constant unification is entirely feasible in a warped 2-brane model.

2. Setup

As in [4], we postulate the presence of a fifth dimension, and an anti-deSitter space metric:

$$ds_5^2 = \frac{1}{k^2 z^2}(dt^2 - dx^2 - dz^2)\,. \tag{2.1}$$

We will generally keep the z-dependence explicit and contract 4D fields with $\eta^{\mu\nu}$. We include two branes: the Planck brane at $z = 1/k$ and the TeV brane at $z = 1/T$. T is related to the size of the extra dimension R by $T = k\exp(-kR)$, and defines the energy scale on the TeV brane. If we take $T \approx$ TeV, we can naturally explain the weak scale in the standard model. The fifth dimension can be integrated out to get an effective four-dimensional theory, valid at energies below T. The effective 4D Planck scale M_{Pl} is given by

$$M_{Pl}^2 = \frac{M^3}{k}\left(1 - \frac{T^2}{k^2}\right). \tag{2.2}$$

It is generally assumed that $T \ll k$ and $k \approx M \approx M_{Pl}$.

Now we put gauge bosons in the bulk. The first question is what is the quantity we wish to calculate. We know we cannot run the coupling on the TeV brane above the strong gravity scale, which is roughly TeV. Furthermore, we are ultimately looking at the zero mode, the only light mode in the theory at energies of order TeV or below. This is important; it means the logarithmic running of the coupling considered in [6, 7] does not apply; it is a result of the sum of all the excited gauge modes, all but one of which are heavy from a low-energy perspective. Therefore, the holographic computation of large threshold corrections does not apply. We are ultimately interested only in the relation of the zero-mode coupling of the four-dimensional theory to the high energy five-dimensional coupling.

3. Effective four-dimensional theory

It is of interest to consider the four-dimensional effective theory. As mentioned above, we will abandon this in favor of the full five-dimensional calculation, but here we discuss the theory and why it is problematic.

The action for a 5D gauge boson is:

$$S_{5D} = \int d^4x dz \sqrt{G}\left[-\frac{1}{4}F_{MN}F^{MN} + \frac{1}{2}\left(a^2 k\delta\left(z - \frac{1}{T}\right) + \tilde{a}^2 k\delta\left(z - \frac{1}{k}\right) + m^2 k^2\right)A^M A_M\right]. \tag{3.1}$$

JHEP11(2001)003

Here a is a mass term on the TeV-brane, $\tilde{a}$ is a mass term on the Planck brane, and m is the bulk mass. The signs are consistent with our metric convention (2.1).

To begin with, we take the bulk boson to be massless, and set $a = m = \tilde{a} = 0$. We expand the 5D bosons in terms of an orthonormal set of KK modes [8]–[12]:

$$A_\mu(x,z) = \sqrt{k}\sum_{j=0} A_\mu^{(j)}(x)\chi_j(z) \tag{3.2}$$

$$A_5(x,z) = \sqrt{k}\sum_{j=1} A_5^{(j)}(x)\frac{1}{m_j}\partial_z\chi_j(z)\,. \tag{3.3}$$

The expansion of A_5 is chosen to diagonalize the couplings between A_5 and A_μ (see below). Keep in mind that the mass dimensions are $[z] = -1$, $[g_{5D}] = -1/2$, $[A_M] = 3/2$, $[A_5^{(n)}] = [A_\mu^{(n)}] = 1$, and $[\chi_n] = 0$. The eigenfunctions satisfy:

$$\partial_z(\frac{1}{z}\partial_z\chi_j) = -\frac{m_j^2}{z}\chi_j, \quad \int \chi_i(z)\chi_j(z)\frac{dz}{z} = \delta_{ij} \tag{3.4}$$

and therefore have the form:

$$\chi_j(z) = z(\mathcal{J}_1(m_j z) + \beta_j \mathcal{Y}_1(m_j z))\,. \tag{3.5}$$

We assume that the 5D boson, and all the KK modes, have even parity under the Z_2 orbifold transformation. Consequently their derivatives must vanish on both boundaries. That is, even parity leads to Neumann boundary conditions. Similarly, odd parity leads to Dirichlet boundary conditions, which we will discuss in more detail later on. This leads to the quantization condition:

$$\beta_j = -\frac{\mathcal{J}_0(m_j/k)}{\mathcal{Y}_0(m_j/k)} = -\frac{\mathcal{J}_0(m_j/T)}{\mathcal{Y}_0(m_j/T)}\,. \tag{3.6}$$

For $m_j \ll k$, $\mathcal{Y}_0(m_j/k)$ blows up, and so the masses are basically determined by the zeros of $\mathcal{J}_0(m_j/T)$. Therefore, they are spaced in energy by approximately πT. This spacing is quite general: it is independent of bulk or boundary mass terms, and of the spin of the bulk field; because $\mathcal{J}_\nu$ oscillates with the same period for any ν, bulk fields will always have excitations of order T.

We chose the conditions (3.2), (3.3) and (3.4) to normalize the kinetic terms and diagonalize the couplings in the effective 4D action:

$$S_{4D} = \int d^4x \left[-\frac{1}{4}(F_{\mu\nu}^j)^2 + \frac{1}{2}(\partial_\mu A_5^j)^2 - \frac{1}{2}m_j^2(A_\mu^j)^2 + m_j(\partial_\mu A_5^j)A^{\mu j}\right]. \tag{3.7}$$

This action is explicitly gauge invariant. Indeed, the 5D gauge invariance

$$A_M \to A_M - \frac{1}{g_{5D}}\partial_M\alpha(x,z) \tag{3.8}$$

JHEP11(2001)003

has its own KK decomposition (we expand $\alpha = \alpha_i \chi_i$):

$$\begin{aligned} A_\mu(x,z) &\to \sqrt{k} A_\mu^i(x)\chi^i(z) - \frac{1}{g_{5D}}\partial_\mu \alpha_i(x)\chi^i(z) \\ &= \sqrt{k}\left[A_\mu^i(x) - \frac{1}{\sqrt{k}g_{5D}}\partial_\mu\alpha_i(x)\right]\chi^i(z) \\ A_5(x,z) &\to \sqrt{k} A_5^i(x)\frac{1}{m_i}\partial_z\chi^i(z) - \frac{1}{g_{5D}}\alpha_i(x)\partial_z\chi^i(z) \\ &= \sqrt{k}\left[A_5^i(x) - \frac{m_i}{\sqrt{k}g_{5D}}\alpha_i(x)\right]\frac{1}{m_i}\partial_z\chi^i(z)\,. \end{aligned}$$

We can plug this back into the action to see that each mode of the 5D gauge field has an independent gauge freedom.

At this point, it is standard to set $A_5 = 0$. We see immediately that this breaks all but the zero mode of the gauge invariance (there is no $A_5^{(0)}$ since $\partial_z\chi_0(z) = 0$). All modes of A_5 are eaten by the corresponding excited modes of the gauge boson. This is the unitary gauge. The Goldstone boson (A_5) is eliminated and the massive gauge boson propagators must take the form:

$$\langle A_5^j(p) A_5^j(-p)\rangle = \frac{-i}{p^2 - m_j^2}\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{m_j^2}\right). \tag{3.9}$$

Although this makes the 4D action look very simple, it is problematic for evaluating loop diagrams. When we do the full 5D calculation later on, we will use the Feynman-'t Hooft gauge, in which A_5 is included as a physical particle.

Because it will be useful for interpreting our results, we present the mass of the lightest KK modes for states of various spin and mass. For the general lagrangian (3.1), with arbitrary mass parameters, the spectrum of a bulk gauge boson is determined by:

$$\begin{aligned} &\frac{(-\tilde{a}^2/2 + 1 - \nu)\,\mathcal{J}_\nu(m_j/k) + m_j/k\,\mathcal{J}_{\nu-1}(m_j/k)}{(-\tilde{a}^2/2 + 1 - \nu)\,\mathcal{Y}_\nu(m_j/k) + m_j/k\,\mathcal{Y}_{\nu-1}(m_j/k)} = \\ &\qquad = \frac{(a^2/2 + 1 - \nu)\,\mathcal{J}_\nu(m_j/T) + m_j/T\,\mathcal{J}_{\nu-1}(m_j/T)}{(a^2/2 + 1 - \nu)\,\mathcal{Y}_\nu(m_j/T) + m_j/T\,\mathcal{Y}_{\nu-1}(m_j/T)}\,. \end{aligned} \tag{3.10}$$

where $\nu = \sqrt{1 + m^2}$. If m or $\tilde{a}$ is nonzero, the lowest mass is of order T. It cannot be lowered below T unless m or $\tilde{a}$ are of the order $T/k \approx 10^{-13}$. For the WZ bosons to pick up a weak scale mass without fine tuning, it must come from the TeV brane. It is not hard to show that for $m = \tilde{a} = 0$ but $a \neq 0$, the lowest mass is given by [9]:

$$m_0 \approx \frac{a}{\sqrt{2kR}}T\,. \tag{3.11}$$

If $T =$ TeV, then we can get the weak scale with $a \approx 0.1$.

JHEP11(2001)003

k/T $= 10^{13}$	massless vector	massive vector $m=1$	massive vector $m=5$	massive vector $m^2=-1$	massless Dirichlet vector	massless scalar	massive scalar $m_s=1$	massless Dirichlet scalar	Dirichlet scalar $m_s=1$
m_0/T	0	2.869	6.873	1.297	3.832	0	4.088	5.136	5.434
m_1/T	2.458	6.086	10.762	4.134	7.016	3.832	7.321	8.417	8.739
m_2/T	5.575	9.249	14.196	7.213	10.174	7.016	10.498	11.620	11.953

Table 1: The exact numerical KK spectrum with $e^{kR} = k/T = 10^{13}$.

To get a feel for how the spectrum depends on various parameters, we present in table 1 the exact numerical KK spectrum with $e^{kR} = k/T = 10^{13}$.

The lagrangian for a massive scalar is $-1/2\sqrt{G}(\partial_M \phi \partial^M \phi - m_s^2 \phi^2)$.

Next, we look the couplings between the KK modes in a non-abelian theory.

$$S \supset \int d^4x \left[-g_{ijk} f^{abc} (\partial_\mu A_\nu^{ia}) A^{j\mu b} A^{k\nu c} - \frac{g_{ijkl}}{4} (f^{eab} A_\mu^{ia} A_\nu^{jb})(f^{ecd} A^{k\mu c} A^{l\nu d}) \right], \quad (3.12)$$

where the coupling constants are given by overlap integrals.

$$g_{ijk} = g_{5D}\sqrt{k} \int \frac{dz}{z} \chi_i \chi_j \chi_k \quad \text{and} \quad g_{ijkl} = g_{5D}^2 k \int \frac{dz}{z} \chi_i \chi_j \chi_k \chi_l \,. \quad (3.13)$$

In flat space, momentum conservation in the fifth dimension implies that KK number is conserved. For example, g_{234} would have to vanish, but g_{224} would not. In curved space this is not true: $g_{ijk} \neq 0$ in general. We can say something for the zero mode, however. Since we have included no masses, its profile is constant and equal to:

$$\chi_0(z) = \frac{1}{\sqrt{kR}} \,. \quad (3.14)$$

Because the χ's are orthonormal and χ_0 is constant:

$$g_{0ij} = \frac{g_{5D}}{\sqrt{R}} \delta_{ij} \quad \text{and} \quad g_{00ij} = \frac{g_{5D}^2}{R} \delta_{ij} \,. \quad (3.15)$$

If we set $g_{5D}^2 = g_{4D}^2 R$, then the effective theory with just the zero mode looks identical to a 4D system. Moreover, the zero mode couples with equal strength to all the KK modes.

It's easy to get a rough idea of how the coupling would run if we took the effective theory at face value. That is, we assume all the KK modes are separate particles, and we use a 4D regularization scheme. At low energy, below $m_1 \approx T$, only the zero mode can run around the loops. As the energy is increased to q, q/T modes are visible. The result is power law running, similar to what has been observed in [2] . This is not the correct result.

One possible improvement, suggested by Pomarol in [5], is to regulate with a Pauli-Villars field with a 5 dimensional mass. This field will have a KK spectrum roughly matching the KK spectrum of the gauge boson, except that it will have

JHEP11(2001)003

a heavy mode near its 5D mass instead of a massless zero mode. Thus all the propagators for the KK modes will roughly cancel and only the zero mode will contribute to running. It will turn out that this is superficially similar to the result we will end up getting from the 5D calculation. However, Pauli-Villars requires that we take the mass of the regulator to infinity, in order to decouple the negative norm states. But an infinitely massive field no longer has a TeV scale KK masses, so it no longer is effective as a regulator. Moreover, it has no hope of telling us threshold corrections, as unitary is violated in the regime where the regulator works.

The root of the problem is that the effective theory breaks down at about a TeV, and so the KK picture is not trustworthy at the high energy scales necessary to probe unification. One can calculate on the Planck brane, but one still has to deal with bulk gauge bosons. The holographic calculation would be at strong coupling. A rigorous perturbative approach is to explore the 5D theory directly.

4. 5D Position/momentum space propagators

To study the 5D theory, we will work in position space for the fifth dimension, but momentum space for the other four.

4.1 R_ξ Gauges

Before gauge fixing, the quadratic terms in the 5D lagrangian are:

$$L = \frac{1}{2kz}\left[A_\mu(\partial^2\eta^{\mu\nu} - z\partial_z(\frac{1}{z}\partial_z)\eta^{\mu\nu} - \partial^\mu\partial^\nu)A_\nu + 2A_5\partial_z\partial^\mu A_\mu - A_5\partial^2 A_5\right]. \quad (4.1)$$

We would like to set $A_5 = 0$ and then choose the Lorentz gauge $\partial_\mu A^\mu = 0$. But these conditions are incompatible. Instead, we will use the following gauge-fixing functional:

$$\Delta L = -\frac{1}{2\xi kz}\left[\partial_\mu A^\mu - \xi z\partial_z\left(\frac{1}{z}A^5\right)\right]^2. \quad (4.2)$$

This produces:

$$L + \Delta L = \frac{1}{2kz}\Bigg[A_\mu\left(\partial^2\eta^{\mu\nu} - z\partial_z\left(\frac{1}{z}\partial_z\right)\eta^{\mu\nu} - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu\right)A_\nu + \\ + A_5(-\partial^2)A_5 + \xi A_5\partial_z\left(z\partial_z\left(\frac{1}{z}A_5\right)\right)\Bigg]. \quad (4.3)$$

We can then read off the equation that the A_μ propagator must satisfy:

$$\langle A^\mu A^\nu\rangle = -iG_p(z,z')\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - iG_{\frac{p}{\sqrt{\xi}}}(z,z')\left(\frac{p^\mu p^\nu}{p^2}\right), \quad (4.4)$$

JHEP11(2001)003

where $p_\mu = i\partial_\mu$ is the four-momentum, and

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + p^2\right] G_p(z,z') = zk\delta(z-z')\,. \tag{4.5}$$

We define A_5's propagator as $\langle A_5 A_5\rangle \equiv i\frac{1}{\xi}G^{1,i}_{\frac{p}{\sqrt{\xi}}}(z,z')$, where

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + \frac{1}{\xi}p^2 + \frac{1}{z^2}\right] G^{1,i}_{\frac{p}{\sqrt{\xi}}}(z,z') = zk\delta(z-z')\,. \tag{4.6}$$

We choose this notation for the following reason. If we had included a bulk mass $\frac{1}{2}m^2k^2A_MA^M\sqrt{G}$ in L, (4.5) would have been

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + p^2 - \frac{m^2}{z^2}\right] G_p^{1,m}(z,z') = zk\delta(z-z')\,. \tag{4.7}$$

Then we can interpret (4.6) as saying that A_5 has the Green's function of a vector boson with bulk mass $m^2 = -1$. For simplicity we will continue to write $G_p = G_p^{1,0}$ for the gauge boson.

We can also work out the ghost lagrangian by varying the gauge fixing functional.

$$L_{\text{ghost}} = \frac{1}{kz}c\left(-\partial_\mu D^\mu - \xi z\partial_z\left(\frac{1}{z}\partial_z\right)\right)c\,. \tag{4.8}$$

Which makes the ghost propagator:

$$\langle cc\rangle = i\frac{1}{\xi}G_{\frac{p}{\sqrt{\xi}}}(z,z')\,. \tag{4.9}$$

Note that the ghosts do not couple to A_5 directly, as they would not couple directly to Goldstone bosons in a conventional spontaneously broken gauge theory.

If we take $\xi = \infty$, we get the unitary gauge. The gauge boson propagator looks like:

$$\langle A^\mu A^\nu\rangle = -iG_p(z,z')\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - iG_0(z,z')\left(\frac{p^\mu p^\nu}{p^2}\right). \tag{4.10}$$

The first term is the transverse polarization states of all the KK modes. We can think of the second term as subtracting off the longitudinal form of the zero mode, Then the zero mode's contribution has just the regular $\eta^{\mu\nu}$ tensor structure. In this gauge, the A_5 and ghost propagators are zero. It is easy to imagine how this gauge would make loop calculations very problematic.

$\xi = 0$ is the Lorentz gauge. Here the A_μ propagator is purely transverse, and A_5 and the ghosts have 4-dimensional propagators:

$$\langle A^\mu A^\nu\rangle = -iG_p(z,z')\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) \qquad \langle A_5A_5\rangle = \frac{i}{Rp^2} \qquad \langle cc\rangle = \frac{i}{Rp^2}\,. \tag{4.11}$$

Finally, $\xi = 1$ is the Feynman-'t Hooft gauge. The propagators are:

$$\langle A^\mu A^\nu\rangle = -iG_p(z,z')\eta^{\mu\nu} \qquad \langle A_5A_5\rangle = iG_p^{1,i}(z,z) \qquad \langle cc\rangle = iG_p(z,z)\,. \tag{4.12}$$

This is the most intuitive gauge. The A_5 supplies the longitudinal polarizations to the excited modes of A_μ.

JHEP11(2001)003

4.2 Solving the Green's functions

To solve (4.5), we first find the homogeneous solution. Defining $u \equiv \min(z, z')$ and $v \equiv \max(z, z')$, this is:

$$G_p(u,v) = u(A\mathcal{J}_1(pu) + B\mathcal{Y}_1(pu)) = v(C\mathcal{J}_1(pv) + D\mathcal{Y}_1(pv)) \tag{4.13}$$

$\mathcal{J}$ and $\mathcal{Y}$ are Bessel functions. For positive parity under the orbifold Z_2, we must impose Neumann boundary conditions at both branes:

$$\partial_u G_p\left(\frac{1}{k}, v\right) = \partial_v G_p\left(u, \frac{1}{T}\right) = 0\,. \tag{4.14}$$

Finally, matching the two solutions over the delta function leads to the fully normalized Green's function:

$$G_p(u,v) = \frac{\pi}{2}\frac{kuv}{AD - BC}(A\mathcal{J}_1(pu)) + B\mathcal{Y}_1(pu))(C\mathcal{J}_1(pv) + D\mathcal{Y}_1(pv)), \tag{4.15}$$

where

$$\begin{array}{lll} A = -\mathcal{Y}_0(p/k) & \text{and} & C = -\mathcal{Y}_0(p/T) \\ B = \mathcal{J}_0(p/k) & \text{and} & D = \mathcal{J}_0(p/T)\,. \end{array} \tag{4.16}$$

If the gauge boson has negative parity under the orbifold Z_2, then it must satisfy Dirichlet boundary conditions at both branes (hence we will call it a Dirichlet boson). Its Green's function will have the same form as (4.15) but with:

$$\begin{array}{lll} A = -\mathcal{Y}_1(p/k) & \text{and} & C = -\mathcal{Y}_1(p/T) \\ B = \mathcal{J}_1(p/k) & \text{and} & D = \mathcal{J}_1(p/T)\,. \end{array} \tag{4.17}$$

Fields of other spin can be derived analogously. For example, the Green's function for a massless scalar is:

$$S_p(u,v) = \frac{\pi}{2}\frac{k^3u^2v^2}{A_sD_s - B_sC_s}(A_s\mathcal{J}_2(pu)) + B_s\mathcal{Y}_2(pu))(C_s\mathcal{J}_2(pv) + D_s\mathcal{Y}_2(pv)) \tag{4.18}$$

with

$$\begin{array}{lll} A_s = -\mathcal{Y}_1(p/k) & \text{and} & C_s = -\mathcal{Y}_1(p/T) \\ B_s = \mathcal{J}_1(p/k) & \text{and} & D_s = \mathcal{J}_1(p/T) \end{array} \tag{4.19}$$

In general, for scalars ($\sigma = 2$), fermions ($\sigma = 1/2$) or vectors ($\sigma = 1$) and with bulk mass m, as in (3.1), the Green's functions are:

$$\begin{aligned} G_p^{\sigma m}(u,v) = \frac{\pi}{2}&\frac{k^{2\sigma-1}u^\sigma v^\sigma}{A_{\sigma m}D_{\sigma m} - B_{\sigma m}C_{\sigma b}} \times \\ &\times(A_{\sigma m}\mathcal{J}_\nu(pu)) + B_{\sigma m}\mathcal{Y}_\nu(pu))(C_{\sigma m}\mathcal{J}_\nu(pv) + D_{\sigma m}\mathcal{Y}_\nu(pv)\,, \end{aligned} \tag{4.20}$$

JHEP11(2001)003

where $\nu = \sqrt{\sigma^2 + m^2}$ and

$$\begin{aligned} A_{\sigma m} &= -\mathcal{Y}_{\nu-1}(p/k) + (\nu - \sigma)\frac{k}{p}\mathcal{Y}_\nu(p/k) \\ B_{\sigma m} &= \mathcal{J}_{\nu-1}(p/k) - (\nu - \sigma)\frac{k}{p}\mathcal{J}_\nu(p/k) \\ C_{\sigma m} &= -\mathcal{Y}_{\nu-1}(p/T) + (\nu - \sigma)\frac{T}{p}\mathcal{Y}_\nu(p/T) \\ D_{\sigma m} &= \mathcal{J}_{\nu-1}(p/T) - (\nu - \sigma)\frac{T}{p}\mathcal{J}_\nu(p/T)\,. \end{aligned} \tag{4.21}$$

Similar results for KK decompositions can be found in [10]. Keep in mind that although A_5 and the ghosts are scalars, their propagators involve the spin-1 Green's function. Intuitively, this is expected because they are necessary for gauge invariance.

We will eventually have to perform a Wick rotation, so that a euclidean momentum cutoff can be imposed on all the components of p_μ. To this end, we will need the Green's functions with $p \to iq$. These functions are still real. It is easiest to get them by re-solving equations like (4.5) with $p^2 \to -q^2$. The result is:

$$\begin{aligned} G_q^{\sigma m}(u,v) = &\frac{k^{2\sigma-1}u^\sigma v^\sigma}{A_{\sigma m}D_{\sigma m} - B_{\sigma m}C_{\sigma m}} \times \\ &\times(A_{\sigma m}\mathcal{K}_\nu(qu)) + B_{\sigma m}\mathcal{I}_\nu(qu))(C_{\sigma m}\mathcal{K}_\nu(qv) + D_{\sigma m}\mathcal{I}_\nu(qv)\,, \end{aligned} \tag{4.22}$$

where $\nu = \sqrt{\sigma^2 + m^2}$ as before and

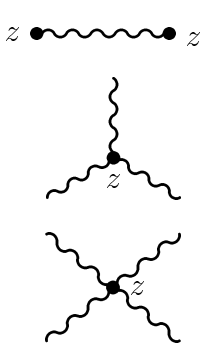
$$\begin{aligned} A_{\sigma m} &= \mathcal{I}_{\nu-1}(q/k) - (\nu - \sigma)\frac{k}{q}\mathcal{I}_\nu(q/k) \\ B_{\sigma m} &= \mathcal{K}_{\nu-1}(q/k) + (\nu - \sigma)\frac{k}{q}\mathcal{K}_\nu(q/k) \\ C_{\sigma m} &= \mathcal{I}_{\nu-1}(q/T) - (\nu - \sigma)\frac{T}{q}\mathcal{I}_\nu(q/T) \\ D_{\sigma m} &= \mathcal{K}_{\nu-1}(q/T) + (\nu - \sigma)\frac{T}{q}\mathcal{K}_\nu(q/T)\,. \end{aligned} \tag{4.23}$$

5. Feynman rules

It is fairly straightforward to derive the Feynman rules in these coordinates. External particles are specified by their 4-momentum and their position in the fifth dimension. The vertices have additional factors of the metric which can be read off the lagrangian. Both loop 4-momenta and internal positions must be integrated over. The Feynman rules for a non-abelian gauge theory are (in the Feynman-'t Hooft

JHEP11(2001)003

gauge):

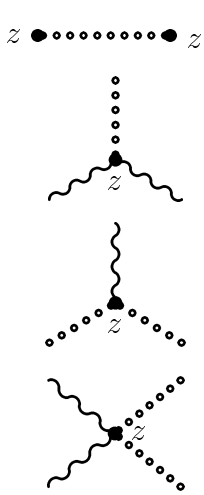


$$= -iG_p(z,z')\eta^{\mu\nu} \tag{5.1}$$

$$= g_{5d}\frac{1}{kz}f^{abc}[\eta^{\mu\nu}(k-p)^\rho + \eta^{\nu\rho}(p-q)^\mu + \eta^{\rho\mu}(q-k)^\nu] \tag{5.2}$$

$$= -ig_{5d}^2\frac{1}{kz}N^{\mu\nu,\rho\sigma}_{abcd} \tag{5.3}$$

where $N^{\mu\nu,\rho\sigma}_{abcd}$ is the standard 4-boson vertex tensor and group structure. Then there are the A_5 contributions:



$$= iG_p^{1,i} \tag{5.4}$$

$$= g_{5d}\frac{1}{kz}f^{abc}(\partial_z^1 - \partial_z^2)\eta^{\mu\nu} \tag{5.5}$$

$$= g_{5d}\frac{1}{kz}f^{abc}(p^\mu - q^\mu) \tag{5.6}$$

$$= g_{5d}^2\frac{1}{kz}\eta^{\mu\nu}(f^{eab}f^{ecd} + f^{ead}f^{ecb}) \tag{5.7}$$

The derivatives ∂_z^1 and ∂_z^2 in (5.5) are to be contracted with the gauge boson lines, while p^μ and q^μ in (5.6) are the momenta of the A_5 lines. There are no 3 or 4 A_5 vertices because of the antisymmetry of f^{abc}. The Feynman rules for other bulk fields can be derived analogously, with due regard for the factors of metric at the vertices. For example, a ϕ^4 vertex would have a factor of $(kz)^{-5}$, while a $\phi^2 A_\mu A^\mu$ vertex would go like $(kz)^{-3}$. Ghosts, which are scalars, technically come from terms compensating for the gauge invariance of $F_{MN}F^{MN}$, so they have $(kz)^{-1}$ vertices.

6. Limits of the Green's functions

Before we evaluate the quantum effects, we will study the propagator in various limits. To do this, we find it convenient to work with euclidean momentum. Recall that the Green's function for the massless vector boson is:

$$G_q(u,v) = kuv\frac{[\mathcal{I}_0(q/k)\mathcal{K}_1(qu) + \mathcal{K}_0(q/k)\mathcal{I}_1(qu)][\mathcal{I}_0(q/T)\mathcal{K}_1(qv) + \mathcal{K}_0(q/T)\mathcal{I}_1(qv)]}{\mathcal{I}_0(q/k)\mathcal{K}_0(q/T) - \mathcal{K}_0(q/k)\mathcal{I}_0(q/T)}. \tag{6.1}$$

JHEP11(2001)003

One advantage of this form is that the modified Bessel functions, $\mathcal{I}$ and $\mathcal{K}$, have limits which are exponentials, while the ordinary Bessel functions oscillate.

The first regime we consider is $q \ll T$:

$$G_{q \ll T}(u, v) \to -\frac{1}{R\, q^2}\,. \tag{6.2}$$

This is what we expect; at low energy, only the zero mode of the gauge boson is accessible. Its profile is constant so G is naturally independent of u and v. The factor of R is absorbed in the conversion from 5D to 4D couplings.

It is also useful to consider the next term in the small q expansion of G_q at a point u in the bulk. This will tell us the size of the 4-Fermi operator which comes from integrating out the excited KK modes.

$$G_0(u, u) = -\frac{k}{4T^2k^2R^2} + \frac{1}{2}u^2\left(k + \frac{1}{R}\right) - \frac{1}{R}u^2 \log ku + \frac{1}{4k^3R^2}\,. \tag{6.3}$$

On the Planck brane and TeV branes respectively, it is:

$$G_0\left(\frac{1}{k}, \frac{1}{k}\right) \approx -\frac{k}{4T^2k^2R^2} \tag{6.4}$$

$$G_0\left(\frac{1}{T}, \frac{1}{T}\right) \approx -\frac{k}{2T^2}\,. \tag{6.5}$$

The additional $(kR)^2$ suppression on the Planck brane over the TeV brane can be understood from the KK picture. The light modes have greater amplitude near their masses, which are near the TeV brane. While there are the same number of heavier modes, localized near the Planck brane, these are additionally suppressed by the square of their larger masses. This has been made quantitative by Davoudiasl et al. in [12]. They calculated the equivalent of G_0 using the KK mode propagators:

$$G_0(z, z) = -k \sum \chi_i(z)^2 \frac{1}{m_i^2} \tag{6.6}$$

and showed how this number is constrained by precision tests of the standard model:

$$V = -M_W^2 R G_0 < 0.0013\,. \tag{6.7}$$

Using our formula, this forces $T > 8.9$ TeV if fermions are on the TeV brane and $T > 197$ GeV if fermions are on the Planck brane (for $kR \approx 32$).

With mass terms, the Green's function satisfies:

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + p^2 - \frac{1}{k^2z^2}\left(a^2k\delta(z - \frac{1}{T}) + \tilde{a}^2k\delta(z - \frac{1}{k}) + m^2k^2\right)\right] G_p^{1,m}(z, z') = \\ = zk\delta(z - z')\,. \tag{6.8}$$

JHEP11(2001)003

We can work through the same analysis as in the massless case. We find that if m or $\tilde{a}$ is not zero, or if a is very large, then the propagator is constant at low momentum. For a non-zero bulk mass:

$$G_0^{1,m}(u,v) = -\frac{kuv}{(ku)^\nu (kv)^\nu} \frac{(1+\nu+(\nu-1)(ku)^{2\nu})(1+\nu+(\nu-1)(Tv)^{2\nu})}{2\nu(\nu^2-1)}\,. \quad (6.9)$$

This tells us the strength of the 4-Fermi operators generated by integrating out heavy fields. For example, if we have a unified model where the X and Y bosons get a bulk mass of order k, then on the Planck and TeV branes ($\nu > 1$):

$$RG_0^{1,m}\left(\frac{1}{k},\frac{1}{k}\right) \approx -\frac{kR}{k^2(\nu-1)} \quad (6.10)$$

$$RG_0^{1,m}\left(\frac{1}{T}.\frac{1}{T}\right) \approx -\frac{kR}{T^2(\nu+1)}\,. \quad (6.11)$$

We know that constraints from proton decay force this number to be smaller than $1/(10^{16}\mathrm{GeV})^2$. In particular, we are safe on the Planck brane if $k > 10^{16}$ GeV for any non-zero bulk mass $m > k/T$. On the TeV brane, however, there is no value of the bulk mass which will sufficiently suppress proton decay; it is suppressed by at most $1/T^2$. We clearly need to prevent this contribution. We discuss this later in the unification section.

Increasing q, we find that for $q \gg T$, but $qu \ll 1$ and $qv \ll 1$:

$$G_q(u,v) \to -\frac{k}{q^2\left(\log\left(2k/q\right)-\gamma\right)}\,, \quad (6.12)$$

where $\gamma \approx 0.577$ is the Euler-Mascheroni constant. This is valid on the Planck brane at $u = v = 1/k$ for $q < k$. In particular, it confirms results of [6, 5] that there is a tree level running of the coupling with q. For $q \gg T$ on the TeV brane,

$$G_q\left(\frac{1}{T},\frac{1}{T}\right) \to -\frac{k}{qT}\,. \quad (6.13)$$

That the propagator goes as $1/q$ instead of $1/q^2$ in this regime is evidence of what we noted in the effective theory: there are q/T effectively massless modes which contribute at energy q. It is also evidence that this propagator is not valid for $q > T$ on the TeV brane.

Next, we look at $q \gg T$ and $qu, qv > 1$. Here the propagator looks like:

$$G_q(u,v) \to -\frac{k\sqrt{uv}}{2q} e^{-q(v-u)}\,. \quad (6.14)$$

The $1/q$ dependence has the same explanation as (6.13). Note that the propagator vanishes unless u and v are nearly coincident in the fifth dimension. Finally, we can consider very large energy, $q \gg k$:

$$G_q(u,v) \to -\frac{k\sqrt{uv}}{q} \frac{\cosh(q(u-1/k))\cosh(q(1/T-v))}{\sinh(q(1/T-1/k))}\,. \quad (6.15)$$

JHEP11(2001)003

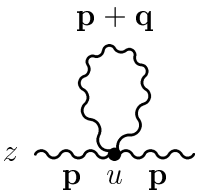
Since we are at energies much higher than the curvature scale, $q \gg k$, we get a result very similar to the propagator in flat space with the fifth dimension bounded at $1/k$ and $1/T$:

$$G_q^{flat}(u,v) \to -\frac{1}{q}\frac{\cosh(q(u-1/k))\cosh(q(1/T-v))}{\sinh(q(1/T-1/k))}\,. \tag{6.16}$$

7. Regulating 5D loops

From studying the 5D propagator in the previous section, we have learned that it cannot be trusted for $qu \gg 1$. This is to be expected if the physical cutoff is around k, since the cutoff on the momentum integral will scale with position in the fifth dimension. So we understand that we need a position-dependent cutoff on four-dimensional momentum. The obvious way to implement this cutoff is to integrate up to momentum $q = \Lambda/(kz)$ at a point z in the bulk. But we will now show that the Green's function must also be renormalized. The correct procedure is to recompute the Green's function at an energy q with boundary conditions from an effective IR brane at $z = \Lambda/(kq)$, and then perform the integral.

Consider the following diagram, which contributes to the gauge boson self energy:



$$\propto g_{5D}^2 \int d^4q \int \frac{du}{ku} G_p(z,u) G_p(u,z') G_{p+q}(u,u) \tag{7.1}$$

We will eventually be concerned with the correction to the zero mode propagator, so we set the external momentum $p = 0$. The low energy propagator is given by (6.2) which is independent of u and v. Since the tree level potential is proportional to $g_5^2/(Rp^2)$ we make the identification $g_{5D}^2 = g_{4D}^2 R$ as before. Also, we will assume the Ward identities are still satisfied and pull out a factor of q^2/p^2. Then the integral reduces to:

$$\frac{1}{Rp^2} g_{4D}^2 \int q dq \int \frac{du}{ku} G_q(u,u)\,. \tag{7.2}$$

The $1/(Rp^2)$ in front corresponds to the tree level propagator we are modifying. That factor of $1/R$ gets absorbed when we cap the ends with a g_{5D}^2 in a full S-matrix calculation.

Later on, we will calculate this integral exactly, but for now, we only which to elucidate the regularization scheme. So for a toy calculation, we will pretend that there is only a zero mode, and so $G_p(u,u)$ has the $p \ll T$ form (6.2) at all energies.

JHEP11(2001)003

First, suppose we have a flat cutoff, at $q = \Lambda$. We know this is wrong, but if we just have the zero mode, it should give precisely the 4D result. Indeed,

$$\int_\mu^\Lambda q dq \int_{1/k}^{1/T} \frac{du}{ku} \frac{1}{q^2 R} = \log\left(\frac{\Lambda}{\mu}\right) \tag{7.3}$$

which is just what we want. Now, suppose we cut off u at $\Lambda/(kq)$, with this Green's function. Then we have:

$$\int_\mu^\Lambda q dq \int_{1/k}^{\Lambda/(kq)} \frac{du}{ku} \frac{1}{q^2 R} = \frac{1}{2kR} \log^2\left(\frac{\Lambda}{\mu}\right) \approx \frac{1}{2} \log\left(\frac{\Lambda}{\mu}\right), \tag{7.4}$$

where we have taken $\mu = \Lambda T/k$ in the last step. Only half the contribution of the zero mode shows up because at an energy q, we are only including $\Lambda T/(kq)$ of it.

Now suppose the IR brane were at $u = \Lambda/(kq)$ instead of $u = 1/T$. Then, at any energy, there would always be an entire zero mode present. The Green's function would not have a sharp cutoff, but would get renormalized with Neumann boundary conditions appropriate to its energy scale. Of course, the physical brane is still at $u = 1/T$, but the Green's function sees the cutoff as an effective brane. With this regularization, our integral is:

$$\int_\mu^\Lambda q dq \int_{1/k}^{\Lambda/(kq)} \frac{du}{ku} \frac{k}{q^2 \log(\Lambda/q)} = \log\left(\frac{\Lambda}{\mu}\right). \tag{7.5}$$

For further illustration, we can work with the full propagator, instead of just the zero-mode approximation, using our new regularization scheme. It is natural to split the integral into two regions, where the propagator can be well-approximated. We can use (6.12) for small qu and (6.14) for large qu. The small region gives, cutting off at $qu = c$:

$$\int_\mu^{ck} q dq \int_{1/k}^{c/q} \frac{du}{ku} \frac{k}{q^2 (\log(2k/q) - \gamma)} \approx \log\left(\frac{ck}{\mu}\right). \tag{7.6}$$

This is the contribution of one gauge boson, although it is not exactly the ground state. The large qu region gives:

$$\begin{aligned} \int_\mu^{ck} q dq \int_{c/q}^{\Lambda/(kq)} \frac{du}{ku} \frac{ku}{2q} + \int_{ck}^{\Lambda} q dq \int_{1/k}^{\Lambda/(kq)} \frac{du}{ku} \frac{ku}{2q} = \\ = \frac{\Lambda - ck}{2k} \log\left(\frac{ck}{\mu}\right) + \frac{\Lambda}{2k} \log\left(\frac{\Lambda}{ck}\right) - \frac{\Lambda - ck}{2k}. \end{aligned} \tag{7.7}$$

This represents, roughly, the additional contribution from the excited modes. In total, there is a log piece, similar to the 4D log but enhanced by a factor of $(\Lambda/k - c)/2$. and a constant piece proportional to Λ. For relatively low values of Λ, the logarithm will dominate, and theory looks four-dimensional. The constant piece

JHEP11(2001)003

contributes to threshold corrections. Later on, when we calculate the β-function exactly in section 10, we will find similar qualitative understanding to this rough analytic approximation.

Our regularization scheme applies just as well to higher-loop diagrams. We can define the Green's function $G_p(u,v)$ as normalized with a brane at $\Lambda/(pk)$, and zero for $v > \Lambda/(kp)$. This automatically implements the cutoff, and we don't have to worry about how to associate the z of a vertex with the momentum of a line.

As a final justification of our regularization scheme, we can look at a renormalization group interpretation through AdS/CFT [13, 14]. It is well known that scale transformations in the CFT correspond to translations in z. But a scale transformation in a quantum field theory is implemented by a renormalization group flow. It follows that integrating out the high-momentum degrees of freedom in the 4D theory should correspond to integrating out the small z region of the 5D theory. Suppose our 5D lagrangian is defined at some scale M. This scale is associated not only with the explicit couplings in the lagrangian, but also the boundary conditions with which we define the propagators. The high energy degrees of freedom are not aware of the region with $z > (\Lambda M/k)^{-1}$, which includes the TeV brane. Therefore, we are forced to normalize the propagators with an effective virtual brane at $\Lambda M/k$. In this way, we derive the low-energy wilsonian effective action in five dimensions. If we follow this procedure down to energies of order TeV, we can then integrate over the fifth dimension to derive the four-dimensional effective theory.

8. Corrections to the radion potential

As a sample calculation, we compute a two-loop contribution to the vacuum energy that determines the radion potential [15, 16]. Consider the following diagram contributing to the vacuum energy of a scalar:

$$\bigotimes u \;\propto \lambda_{5D} \int_0^{\Lambda T/k} q^3 dq \int_0^{\Lambda T/k} p^3 dp \int_{1/k}^{1/T} \frac{du}{(ku)^5} S_q(u,u) S_p(u,u)\,.$$

Here λ_{5D} is the 5D ϕ^4 coupling. which has dimensions of length. It is related to the 4D coupling by $\lambda_{4D} = \lambda_{5D}(2k^3/(k^2 - T^2)) \approx 2k\lambda_{5D}$. We have cut off the momenta at $q = \Lambda T/k$. The region of integration with q or p greater than $\Lambda T/k$ has no T dependence, and hence cannot contribute to stabilizing the extra dimension. At low energies,

$$S_q(u,u) \approx -\frac{2k}{q^2} + ku^2 - \frac{k^3}{4}u^4 - \frac{\log(k/T)}{k} + \mathcal{O}(q^2)\,. \tag{8.1}$$

Note the enhanced u dependence of the scalar over the vector propagator (compare (6.3)). This expansion, which is quite a good approximation of the full propagator

JHEP11(2001)003

in the region of integration we are considering, gives a vacuum energy (ignoring the numerical constants):

$$V \approx \lambda_{4D}\lambda^4 T^4 + \mathcal{O}(k^{-2}T^6 \log T)\,. \tag{8.2}$$

This expression has the same T-dependence as zero point energy presented in [17].

9. Gauge boson self-energy

In order to address the question of unification, we will now look at how various bulk fields contribute to the 1-loop β-functions. There are 6 diagrams that contribute at 1-loop: 2 gauge boson, 1 ghost, and 3 involving A_5. One of these diagrams is particularly ugly, involving ∂_z acting on the A_μ propagator. After fixing the gauge, evaluating, and summing all these diagrams, we should get a correction to the gauge boson propagator which is transverse. But there is an easier way: the background field method. The idea is to compute the effective action directly, which at 1-loop only involves evaluating functional determinants. Furthermore, we have the freedom to choose the external field to be whatever we like. In particular, we can choose it to be the piece of A_μ which is independent of z. We will see that the quantum fields for A_μ, A_5 and ghosts effectively decouple. The Ward identities will be explicitly satisfied, as the diagrams containing each type of particle will separately produce a transverse correction to the A_μ propagator.

9.1 Background field lagrangian

First, we separate the gauge field into a constant external piece and a fluctuating quantum piece:

$$A_M^a \to \frac{1}{g_{5D}}\left(A_M^a + \mathcal{A}_M^a\right). \tag{9.1}$$

We have also renormalized out the coupling. Note that A_M now has mass dimension 1 as in four-dimensions. If we let D_M be the covariant derivative with respect to A_M only then

$$F_{MN}^a \to F_{MN}^a + D_M\mathcal{A}_N^a - D_N\mathcal{A}_M^a + [\mathcal{A}_M, \mathcal{A}_N]^a. \tag{9.2}$$

We also must take our gauge fixing functional to be A_M-covariant:

$$\Delta L = -\frac{1}{2g^2\xi kz}G(\mathcal{A})^2 = -\frac{1}{2g^2\xi kz}\left[D^\mu\mathcal{A}_\mu^a - \xi z D_z\left(\frac{1}{z}\mathcal{A}_5^a\right)\right]^2. \tag{9.3}$$

JHEP11(2001)003

The lagrangian is then:

$$\begin{aligned} L = & -\frac{1}{4g^2kz}(F^a_{\mu\nu} + D_\mu \mathcal{A}^a_\nu - D_\nu \mathcal{A}^a_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu]^a)^2 + \\ & + \frac{1}{2g^2kz}(F^a_{\mu 5} + D_\mu \mathcal{A}^a_5 - D_z \mathcal{A}^a_\mu + [\mathcal{A}_\mu, \mathcal{A}_5]^a)^2 - \\ & - \frac{1}{2g^2\xi kz}\left[D^\mu \mathcal{A}^a_\mu - \xi z D_z\left(\frac{1}{z}\mathcal{A}^a_5\right)\right]^2 . \end{aligned} \tag{9.4}$$

At 1-loop, we only need to look at terms quadratic in the quantum fields $\mathcal{A}_M$. After an integration by parts, the quadratic lagrangian is:

$$\begin{aligned} L_2 = & -\frac{1}{2g^2kz}\Bigg(\mathcal{A}^a_\mu\left[-(D^2)^{ab}\eta^{\mu\nu} + (D^\nu D^\mu)^{ab}\eta^{\mu\nu} - \frac{1}{\xi}(D^\mu D^\nu)^{ab}\right]\mathcal{A}^b_\nu + \\ & + F^{a\mu\nu}[\mathcal{A}_\mu, \mathcal{A}_\nu]^a\Bigg) + \frac{1}{2g^2kz}\mathcal{A}^a_5\left(-(D^2)^{ab}\right)\mathcal{A}^b_5 + \\ & + \frac{\xi}{2g^2kz}\mathcal{A}^a_5 D_z\left(zD_z\left(\frac{1}{z}\mathcal{A}^b_5\right)\right) - \frac{1}{g^2kz}\mathcal{A}^a_5\left[D_\mu D_z - D_z D_\mu\right]^{ab}\mathcal{A}^b_\mu - \\ & - \frac{1}{2g^2k}A_\mu D_z\left(\frac{1}{z}D_z A_\mu\right) + \frac{1}{g^2kz}F^{a\mu 5}[\mathcal{A}_\mu, \mathcal{A}_5]^a . \end{aligned} \tag{9.5}$$

We derive the ghost lagrangian from variations of $G(\mathcal{A})$:

$$\frac{\delta G}{\delta \alpha} = D^\mu(D_\mu + f^{abc}\mathcal{A}^b_\mu) - \xi z D_z\left[\frac{1}{z}(D_z + f^{abc}\mathcal{A}^b_5)\right] . \tag{9.6}$$

Combining (9.5) with the ghost lagrangian, using the relation

$$\mathcal{A}^a_M[D_R, D_S]^{ab}\mathcal{A}^b_N = -F^a_{RS}[\mathcal{A}_M, \mathcal{A}_N]^a \tag{9.7}$$

the final quadratic lagrangian for the pure non-abelian gauge theory in AdS_5 is:

$$\begin{aligned} L_2 = & -\frac{1}{2g^2kz}\Bigg(\mathcal{A}^a_\mu\left[-(D^2)^{ab}\eta^{\mu\nu} + \left(1-\frac{1}{\xi}\right)(D^\mu D^\nu)^{ab}\right]\mathcal{A}^b_\nu + z\mathcal{A}^a_\mu D_z\left(\frac{1}{z}D_z\mathcal{A}^b_\nu\right)\eta^{\mu\nu} + \\ & + 2F^{a\mu\nu}[\mathcal{A}_\mu, \mathcal{A}_\nu]^a\Bigg) + \frac{1}{2g^2kz}\left(\mathcal{A}^a_5(-D^2)^{ab}\mathcal{A}^b_6 + \xi\mathcal{A}^a_5 D_z\left(zD_z\left(\frac{1}{z}\mathcal{A}^b_5\right)\right)\right) + \\ & + \frac{1}{g^2kz}\bar{c}^a\left[-(D^2)^{ab} + \xi z D_z\left(\frac{1}{z}D_z\cdot\right)\right]c^b . \end{aligned} \tag{9.8}$$

Observe that the cross terms between $\mathcal{A}_\mu$ and $\mathcal{A}_5$ vanish, as expected. At this point, we will specialize to the β function calculation we are interested in. It involves an external zero mode of A_μ, whose profile is constant in the fifth dimension. We simply the external field to be the piece of the original field which is independent of z. We

JHEP11(2001)003

also set $A_5 = 0$. as there is no external A_5 component. This lets us write ∂_z instead of D_z. Then in the Feynman gauge, $\xi = 1$, the lagrangian is:

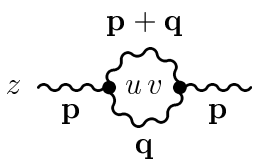
$$\begin{aligned} L_2 = & -\frac{1}{2g^2kz}\left(\mathcal{A}^a_\mu[-(D^2)^{ab}\eta^{\mu\nu}]\mathcal{A}^b_\nu + z\mathcal{A}^a_\mu D_z\left(\frac{1}{z}D_z\mathcal{A}^b_\nu\right)\eta^{\mu\nu} + 2F^{a\mu\nu}[\mathcal{A}_\mu,\mathcal{A}_\nu]^a\right) + \\ & +\frac{1}{2g^2kz}\left(\mathcal{A}^a_5(-D^2)^{ab}\mathcal{A}^b_6 + \mathcal{A}^a_5 D_z\left(zD_z\left(\frac{1}{z}\mathcal{A}^b_5\right)\right)\right) + \\ & +\frac{1}{g^2kz}\bar{c}^a\left[-(D^2)^{ab} + zD_z\left(\frac{1}{z}D_z\cdot\right)\right]c^b\,. \end{aligned} \tag{9.9}$$

We see immediately that the fields have the propagators we derived before (4.12), in the Feynman-'t Hooft gauge. There are no cross terms between A_μ and A_5 and we can evaluate the functional determinant for each field independently. In particular, A_5 is seen as a scalar field transforming in the adjoint representation of the gauge group, with the same Green's function as a vector with bulk mass $m^2 = -1$.

9.2 Functional determinants

We can now evaluate the functional determinants using standard textbook techniques [18]. There are two diagrams which contribute, one is spin-dependent (from the $F^{a\mu\nu}$ vertex in (9.9)) and vanishes for scalars, and the other is spin-independent. (The third diagram from the quartic interaction does not contribute as $d \to 4$ in dimensional regularization, so we will ignore it for simplicity.) Both diagrams are independently transverse in the external momentum. This is evidence that the Ward identity for the 5D gauge invariance is working.

The spin-dependent diagram gives:



$$= -\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(-p)A^b_\mu(p)\int\frac{d^4q}{(2\pi)^4}\frac{p^2\eta^{\mu\nu}-p^\mu p^\nu}{q^2(p+q)^2}4C_rC(j)\times$$

$$q^2(p+q)^2 2\int_{1/k}^{\Lambda/(kq)}\frac{du}{ku}\int_u^{\Lambda/(kq)}\frac{dv}{kv}G_q(u,v)G_{q+p}(u,v) \tag{9.10}$$

where $C(j) = 2$ for vectors and zero for scalars and C_r is Dynkin index for the appropriate representation. Since we are interested in the vacuum polarization in the $p \to 0$ limit, and the transverse projector is already manifest, we can simply set $p = 0$ in the uv integrals. Now change variables to $y = qu$ and $z = qv$ and set $k = 1$. Then the second line above becomes

$$I(\Lambda, q) = q^4 2\int_q^\Lambda\frac{dy}{y}\int_y^\Lambda\frac{dz}{z}G_q\left(\frac{y}{q},\frac{z}{q}\right)^2. \tag{9.11}$$

JHEP11(2001)003

The point of doing this is that the integrand now contains the square of:

$$G_q\left(\frac{y}{q},\frac{z}{q}\right)=\frac{yz}{q^2}\frac{[\mathcal{I}_0(q)\mathcal{K}_1(y)+\mathcal{K}_0(q)\mathcal{I}_1(y)][\mathcal{I}_0(\Lambda)\mathcal{K}_1(z)+\mathcal{K}_0(\Lambda)\mathcal{I}_1(z)]}{\mathcal{I}_0(q)\mathcal{K}_0(\Lambda)-\mathcal{K}_0(q)\mathcal{I}_0(\Lambda)}\,. \quad (9.12)$$

The $1/q^4$ in G_q^2 cancels the q^4 prefactor in (9.11), leaving a dimensionless number multiplying the standard 4D integral. This is not strictly true, as $I(\Lambda,q)$ still has a weak dependence on q. But quite generally, we can write $I(\Lambda,q)=I_0(\Lambda)+I_1(\Lambda)\frac{q}{k}+\cdots$, so for $q\ll k, I=I_0$ is a fine approximation. Anyway, the background field method at 1-loop cannot give us reliable information about additional divergences, or threshold corrections. The best we can do is to use the degree to which I is not constant as a rough measure of the size of the additional corrections.

The other diagram, which is spin-independent. contributes:

$$-\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(-p)A^a_\mu(p)\int\frac{d^4q}{(2\pi)^4}\frac{(2q+q)^\mu(2q+p)^\nu}{q^2(p+q)^2}C(r)d(j)\times$$
$$\times q^2(p+q)^2 2\int_{1/k}^{\Lambda/(kq)}\frac{du}{ku}\int_u^{\Lambda/q}\frac{dv}{kv}G_q(u,v)G_{q+p}(u,v)\,, \quad (9.13)$$

where $d(j)$ is the number of spin components. The second line is exactly the same integral expression, $I(\Lambda,q)$, as before. While the tensor structure of the first line is not explicitly transverse in the external momentum, it is in fact transverse after the q integral is performed (in dimension regularization as $d\to 4$).

Each particle will have a different value for I_0. We must replace G_q with the appropriate propagator (cf. section 4.2) for each particle and redo the integrals in each case. We shall call the result $I_0^{\sigma,m}$, corresponding to $G_p^{\sigma,m}$ from section 4.2. Observe that as with the R_ξ gauges, A_5 and the ghosts have kinetic terms corresponding to spin 1, so they will both have $\sigma=1$. In particular, I_0 for ghosts is identical to I_0 for A_μ.

The reason these diagrams are relevant is that they directly produce F^2 terms in the effective action. Indeed, the Fourier transform of the quadratic terms in F^2 is:

$$-\frac{1}{4g^2}\int d^4x\frac{dz}{kz}F_{\mu\nu}F^{\mu\nu}=-\frac{R}{2g^2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(-p)A^b_\nu(p)(p^2\eta^{\mu\nu}+p^\mu p^\nu)\,. \quad (9.14)$$

Since A_μ is independent of z, we have performed the z-integral explicitly. So we see that we have calculated a correction to the dimensionless coupling $g_{5D}R^{-\frac{1}{2}}$. Equivalently, we have calculated the running of g_{5D} itself, once we absorb the factor of R into the coefficient of the logarithm. The result for the 1-loop β-function is:

$$\beta(g_{5D})=-\frac{g_{5D}^3}{4\pi^2}\frac{1}{R}C_2(G)\left(\frac{11}{3}I_0^{1,0}-\frac{1}{6}I_0^{1,i}\right)+matter\,. \quad (9.15)$$

It may appear that we have calculated the running for only the zero mode of the 5D gauge boson. But gauge invariance implies that there can only be one 5D coupling,

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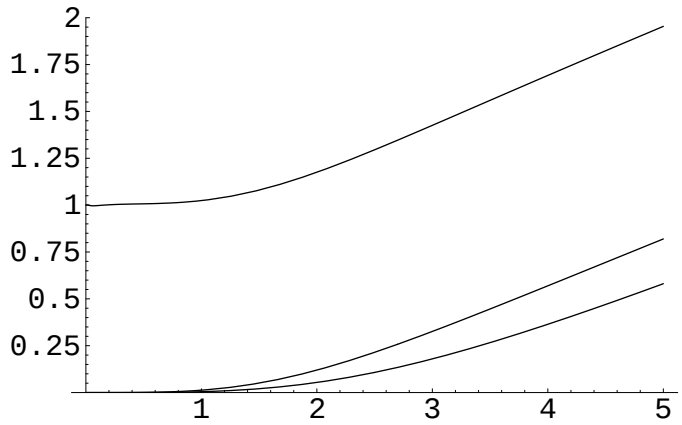

Figure 1: I_0 as a function of Λ/k: from top to bottom: massless vector, massive vector ($m = 1$), and Dirichlet vector.

at any energy scale. So we have in fact calculated the β function of every mode. If we define $g_{4D} \equiv g_{5D} R^{-\frac{1}{2}}$, then the 4D β-function is:

$$\beta(g_{4D}) = -\frac{g_{4D}^3}{4\pi^2} C_2(G) \left(\frac{11}{3} I_0^{1,0} - \frac{1}{6} I_0^{1,i} \right) + matter\,. \tag{9.16}$$

Note that the sign of the A_5 contribution is opposite to that of the ghosts (which contribute $\frac{1}{3} I_0^{1,0}$ to the above expression). This is because they have opposite statistics which changes the sign of the exponent of the functional determinant. It is easy to understand this result; the ghosts remove the two unphysical polarizations of A_μ, and A_5 adds back one of them.

9.3 Numerical results for $I(\Lambda, q)$

The function $I_0(\Lambda)$ is shown in figure (1), and $I(\Lambda, q)$ is shown in figures 2 and 3. Since we need need to perform a Wick rotation to evaluate the 4D integrals, we used the euclidean propagators in calculating $I(\Lambda, q)$. Of course, the constant piece, I_0, is independent of $q \to iq$, and we have confirmed this numerically. It turns out that the integrals converge faster in euclidean space. We can see from figure 1 that I_0 is roughly proportional to the number of KK modes running around the loop. Because of our boundary conditions, the effective spacing between the KK modes at an energy q is $\pi q k/\Lambda$. For the massless 5D vector, at energy q there is a massless mode, plus approximately $q/(\pi q k/\Lambda) = \Lambda/(\pi k)$ other modes visible. This fits roughly with figure 1. The discrepancy is due to the fact that the spacing is not precisely π for the lowest modes, and that the sum of the higher modes is not completely negligible. Notice that the I_0's of massive and Dirichlet cases, which have no massless zero mode, are about 1 less than I_0 for the massless vector. When Λ is bigger than k, and the *beta*-function is correspondingly higher, the unification scale will be lower. A theory of accelerated unification was also considered in ref. [19] in a different scenario.

JHEP11(2001)003

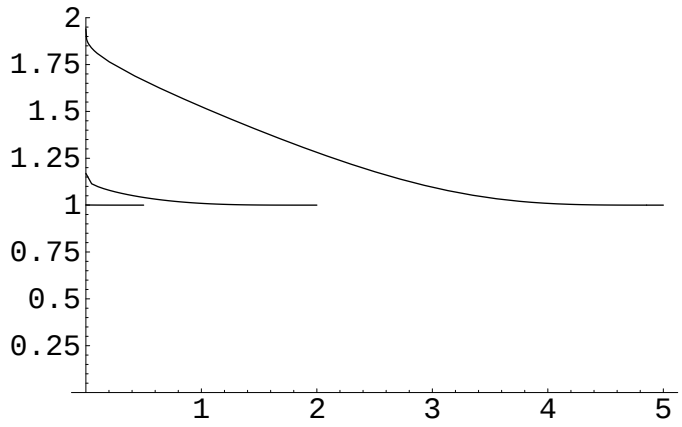

Figure 2: $I(\Lambda, q/k)$ as a function of q/k with Λ fixed. From top to bottom, $\Lambda = 5k, 2k$, and $0.5k$.

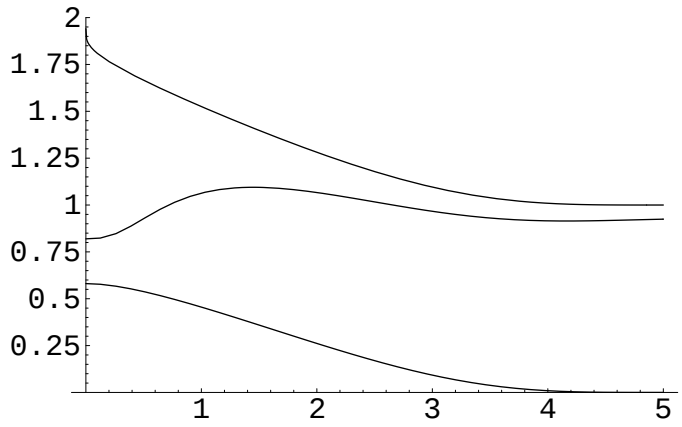

Figure 3: $I(\Lambda = 5k, q/k)$. From top to bottom: massless vector, massive vector (with bulk mass $m = 1$), and Dirichlet vector.

Figures 2 and 3 can be understood similarly. As $q \to k$, the branes approach each other. For the massless case, there is always one complete mode, the zero mode. In this limit the theory looks 4-dimensional. For the massive case, the zero mode exists as well, and so it approximates the massless case. With Dirichlet conditions, the zero mode is eliminated, so the function goes to zero. Note that if we had left the boundary conditions at $1/T$, this function would have gone to zero as $q \to \Lambda$ for any of the cases. We list the values of $I_0(\Lambda)$ for various cases in table 2.

Recall that in the Feynman gauge, ghosts and A_5 get the I_0 of vectors and A_5 has an effective mass $m = 1$. The numbers in this table can be predicted approximately from the KK masses in table 1. We can see that $I_0(\Lambda)$ is approximately the number of modes with mass less than $\Lambda T/k$. This gives us a very useful intuition for seeing how changing the field content affects unification, as we will now illustrate.

JHEP11(2001)003

$I_0^{\sigma,m}(\Lambda)$	massless vector	massive vector $m=1$	massive vector $m=5$	massive vector $m^2=-1$	massless Dirichlet vector	massless scalar	massive scalar $m=1$	massless Dirichlet scalar	Dirichlet scalar $m=1$
$\Lambda=0.5$	1.007	0.001	0.000	0.018	0.000	1.000	0.000	0.000	0.000
$\Lambda=1$	1.024	0.013	0.001	0.147	0.005	1.005	0.004	0.002	0.001
$\Lambda=5$	1.954	0.820	0.178	1.411	0.581	1.581	0.525	0.353	0.315

Table 2: Values of $I_0(\Lambda)$ for fields of various spin and mass.

10. Coupling constant unification

In order to study coupling constant unification, we need to choose a particular model. Because the main motivation of this work is to solve the hierarchy problem using the warp factor, all weak-scale masses should generated from Higgs scalars confined to the TeV brane. We will consider three possible scenarios. The first is that there is no unified group. Indeed, the generic prediction of fundamental theories is only that there should be one coupling constant at high energy, not that there should be a unified group. We put the 3-2-1 gauge bosons in the bulk, and the Higgs and fermions on the TeV brane. From the CFT point of view, the TeV brane fields are to be viewed as condensates. So we can expect there to be a number of bulk fermions or scalars transforming as electroweak doublets, which condense to form the Higgs. In this case, We have to assume there is at a fundamental level a reason to assume the U(1) is normalized in a way consistent with a GUT model. For this, additional physics assumptions are necessary.

A second possibility is that there is a unified group, such as SU(5). If the doublet Higgs is part of a larger multiplet, such as a vector **5**, then the triplet will necessarily have TeV scale excitations, leading to proton decay. This is the standard doublet-triplet problem [20]. One 4D solution is to couple the triplet to a missing partner, which gives it a large mass and decouples it from the standard model. However, this solution will not work with a TeV brane triplet, because its mass can be at most TeV. Instead, one can for example implement the pseudoGoldstone boson method, as in [20, 21]. Briefly, the idea is to postulate a weakly gauged global symmetry, such as SU(6) $\times$ SU(6). This is broken by an adjoint Σ and two fundamentals H and $\bar{H}$. The doublet Higgs arises as a pseudoGoldstone boson, and there is no triplet at all. This sort of condensation also seems likely from the CFT point of view, where all TeV brane fields are composites.

Dimension 6 operators that violate baryon number pose a potential problem. As mentioned before, the X and Y bosons should not couple on the TeV brane. This can be done by having the gauge symmetry not commute with the orbifold transformation [22, 23] so that the WZ have positive parity under Z_2, but the XY have negative parity and have vanishing amplitude on the TeV brane. An additional baryon number symmetry should be imposed on the brane to forbid dangerous operators. Notice that the U(1) on the brane might have a kinetic term, and therefore a coupling, not

JHEP11(2001)003

determined by unification. If this is the case, one would hope the brane couplings for U(1), SU(2), and SU(3) are all big so that a mechanism such as the one in [24] would apply.

A third possibility is that we don't use the TeV brane to generate the weak scale, as in [5]. The hierarchy problem must be solved some other way, such as using supersymmetry.

Now return to the first scenario, with no unified group. The TeV-brane particles will contribute to running only up to $q = \Lambda T/k$. After this, they contribute like the bulk fields which represent their preonic constituents in the CFT. There are many possibilities for what these can be, but for the sake of illustration, we will assume they are either fermions or scalars which have the 3-2-1 quantum numbers of the standard model Higgs. Then the 1-loop β-functions lead to the following running:

$$\alpha_1^{-1}(M_{GUT}) = \alpha_1^{-1}(M_Z) - \frac{2}{\pi}\left(\frac{n_g}{3} + \frac{3}{5}\frac{n_s}{24}\right)\log\left(\frac{M_{GUT}}{M_Z}\right) \tag{10.1}$$

$$\alpha_2^{-1}(M_{GUT}) = \alpha_2^{-1}(M_Z) - \frac{2}{\pi}\left(-\frac{11}{6}I_0^{1,0}(\Lambda) + \frac{1}{12}I_0^{1,i}(\Lambda) + \frac{n_g}{3} + \frac{n_s}{24}\right)\log\left(\frac{M_{GUT}}{M_Z}\right) \tag{10.2}$$

$$\alpha_3^{-1}(M_{GUT}) = \alpha_3^{-1}(M_Z) - \frac{2}{\pi}\left(-\frac{11}{4}I_0^{1,0}(\Lambda) + \frac{1}{8}I_0^{1,i}(\Lambda) + \frac{n_g}{3}\right)\log\left(\frac{M_{GUT}}{M_Z}\right). \tag{10.3}$$

The $I_0^{1,i}$ terms come from the contribution of A_5 which has an effective bulk mass $m = 1$. $I_0(\Lambda)$ is roughly equal to the number of KK modes with mass below $\Lambda T/k$. It is defined exactly in the previous section. There are additional terms in the above equations proportional to $I_1(\Lambda)\frac{M_{GUT}}{k}$, which we will assume to be small. For $I_0 = 1$ (which occurs as $\Lambda \to 0$), these are just the standard equations for 4 dimensional running. For bulk scalars, the effect is $n_s \to n_s I_0^{2,0}(\Lambda)$. If the bulk preons are n_f majorana fermions, which would prevent them from picking up a large bulk mass, then this term should be $n_s \to 2n_f I_0^{1/2,0}(\Lambda)$, where $I_0^{1/2,0}(\Lambda)$ comes from 5D massless fermion loops, and the factor of 2 is because fermions contribute twice as much as complex scalars to the gauge boson self-energy. The same modifications should be made for the n_g term, but these do not affect unification, so we will ignore them.

To show that unification can be improved, we pick a specific model. We choose $\Lambda = 1$ and put 4 majorana fermions in the bulk. We leave $n_g = 3$, to facilitate the comparison with the standard model. Then we use the numerical values: $I_0(1) = 1.024$, $I_0^{1,1}(1) = 0.013$, $I_0^{1/2,0}(1) = 1.009$. We will use the observed values [25] of $\alpha_3(M_Z) = 0.1195$, $\alpha_e^{-1}(M_Z) = 127.934$, and $\sin^2\theta_w = 0.23107$ at the Z-boson mass $M_Z = 91.187$ GeV. The couplings are shown for this case in figure 4. The standard model is shown for comparison.

As Λ is increased, $I_0(\Lambda)$ and $I_0^{1/2,0}(\Lambda)$ grow at roughly the same rate. The net effect is that the β-functions basically scale uniformly with Λ. This will not have much of an effect on whether unification occurs, but it can drastically change the

JHEP11(2001)003

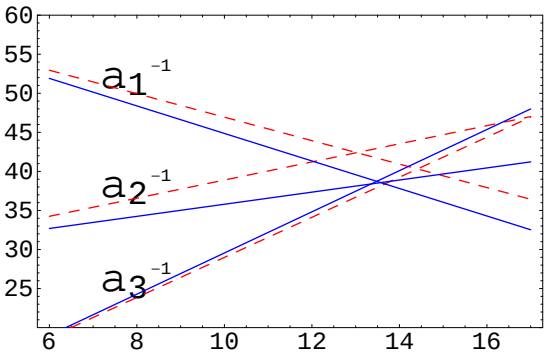

Figure 4: α^{-1} as a function of $\log_{10}(M_{GUT}/M_Z)$. Unification of couplings for $\Lambda = k$ (solid lines). The standard model is shown for comparison (dashed lines).

scale, For example, if we take $\Lambda = 5$, the scale drops from 10^{14} to 10^8. So, if we expect unification near the string scale, we must have $\Lambda \approx 1$. We assumed that $I(\Lambda, q)$ was constant. As we mentioned before, the additional effect from the first order term, I_1 is suppressed by M_{GUT}/k. So if $M_{GUT} \ll k$ it is negligible, but if $M_{GUT} \approx k$, it can be significant. Even though our regularization scheme cannot tell us the precise effect from the 1-loop calculations, we can easily determine the sign. $I_1(\Lambda)$ is the the slope of the curves in figure 2, and is always negative. So for $M_{GUT} \approx k$, these corrections will lower the unification scale.

Now consider the second scenario, where the XY bosons are decoupled from the standard model by changing their Z_2 parity. Then the coefficient of the log picks up an additional piece, proportional to $I_0^d(\Lambda)$, as listed in the massless Dirichlet vector column of table 2. Since complete multiplets do not contribute to unification, we can simplify equations (10.1)–(10.3) by substituting:

$$I_0(\Lambda) \to I_0(\Lambda) - I_0^d(\Lambda)\,. \tag{10.4}$$

The main effect of this is that it allows us to go to higher values of Λ without lowering the unification scale too much. For example, $I_0(5) = 1.954$, but $I_0(5) - I_0^d(5) = 1.473$. This makes $M_{GUT} \approx 10^{11}$ rather than 10^8 as it would be without these additional states. We can also put in fields transforming as adjoints or fundamentals under the GUT group with Dirichlet or Neumann components. There are too many possibilities for us to examine them here, but it is fairly straightforward to work out how they affect unification.

Finally consider the third scenario, where matter is on the Planck brane. Here SU(5) might be broken by a massive adjoint in the standard way, and the triplet might be coupled to some heavy missing partners. Proton decay is suppressed by at least k^{-2}, as we can see from (6.10). Unification is similar to the second scenario, but we must make the replacement $I_0(\Lambda) \to I_0(\Lambda) - I_0^{1,m}(\Lambda)$ in equations (10.1)–(10.3). From table 2, we can see that if the XY bulk mass is $m = 1$, the relevant

JHEP11(2001)003

value is $I_0(5) - I_0^{1,1}(5) = 1.134$. This yields $M_{GUT} \approx 10^{13}$ even for Λ as big as $5k$. However, if the bulk mass of X and Y is too large, for example $m = 5$, then $I_0(5) - I_0^{1,5}(5) = 1.776$, which leads to $M_{GUT} \approx 10^9$. It is clear that there is a lot of room for detailed model building, which we leave for future work.

11. Conclusions

We have shown how to consistently perform Feynman diagram calculations in five-dimensional anti-de Sitter space. Our regularization scheme is inspired by AdS/CFT duality [26, 27, 28]. There we see that scale transformations in the 4D theory are equivalent to z-translations in the 5D theory. Therefore, we can understand how following the renormalization group flow down to the scale μ corresponds to integrating out the fifth dimension from $z = 1/k$ up to $z = \Lambda/(\mu k)$. The correct implementation of this is to renormalize 5D propagators as if the IR brane were at the relevant energy scale for the computation. Not only does this ensure that at a position z the UV cutoff is mediated by the warp factor, but also that a complete 4D mode of the bulk field is always present.

The original Randall-Sundrum scenario was presented as a solution to the hierarchy problem. It is now clear that it is also consistent with coupling constant unification. With standard model matter confined to the TeV brane, the maximum unification scale is naively seen to be $\Lambda T/k$. From the CFT picture, we know this cannot be true. Now we understand how higher scales are reached in 5D as well. We have briefly described some possible unification scenarios. None of them are perfect, and more detailed model building is called for, but it is clear that unification can be improved from the standard model. The key is that even though gauge bosons are in the bulk, running is effectively four dimensional.

Although we originally intended to tie up a loose end of the AdS/CFT picture, it seems like we have revealed a whole new tangle. There are many directions to go from here, and the work is to a large degree unfinished. There are many threshold corrections that we have not yet included. These include subleading terms in the background field calculation, subleading terms in $I(\Lambda, q)$, and a higher loop calculation. Furthermore, the answer depends on the details of the model; here it is not only a question of the GUT group, but also the bulk fields that yield the TeV brane matter. It is well known that while supersymmetric GUTs appear to unify beautifully at 1-loop, at 2-loops unification does not occur within experimental bounds (without involved model building). It is important to see in more detail how well unification works in this model. Finally, we have been somewhat lax about the relationships among the various scales in the theory, namely $M_{GUT}, k, \Lambda k, M, M_{Pl}, R$ and the string scale. These should ultimately be incorporated. Of course, we would also want to motivate Λ in a particular model, and furthermore understand the origin of unification and its scale at a fundamental level.

JHEP11(2001)003

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